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Linear coupling of electromagnetic waves in gyrotropic media

A. G. Shalashov and E. D. Gospodchikov
Institute of Applied Physics, Russian Academy of Sciences, 46 Ulyanova Street, 603950 Nizhny Novgorod, Russia

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The theory of linear mode conversion near evanescent layers is generalized for non-one-dimensionally inhomogeneous gyrotropic media. Effects are found that cannot be described within the standard one-dimensional paradigm of wave tunneling through the evanescent region when the mode coupling region assumes essentially two-dimensional topology.

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Linear coupling of normal waves propagating in weakly inhomogeneous media is a fundamental process important in plasma physics, crystal optics, electrodynamics of metamaterials, etc. In plasma physics, studies of the linear mode coupling were initiated by Ginzburg in his works on "tripling" of radio signals reflected from the Earth's ionosphere [1]. In astrophysics, the linear mode coupling provides radiation escape from dense plasmas, for example, in coronas of stars [2]. In recent years, interest in linear conversion has increased appreciably in connection with high-frequency plasma heating and diagnostics in spherical tokamaks and optimized stellarators in which central regions of a plasma column are poorly accessible for electromagnetic waves launched from vacuum [3].

It has been understood that the classical one-dimensional approach may be insufficient to describe the linear conversion in applications. In this paper we consider the linear coupling of waves propagating in a three-dimensionally inhomogeneous medium. The problem of the multidimensional mode coupling has already received attention. In particular, Cairns and Lashmore-Davies [4] and Kaufman and Freidland [5] inspired a lot of publications based on the "local coupling" approach [6]. In these papers the problem was reduced to two linear wave equations coupled via a scalar constant, $\hat{D}_1E_1=-\eta E_2$ and $\hat{D}_2E_2=-\eta^*E_1$, with $\hat{D}_{1,2}$ being differential wave operators. Within this model two propagating modes are separated by an evanescent layer of a finite width proportional to $|\eta|$; thus the mode conversion appears as tunneling of radiation throughout the evanescent region. Therefore, the coupling efficiency depends exponentially on the width of the evanescent layer exactly in the same way as in the standard *one-dimensional* problem of tunneling through a potential barrier in quantum mechanics. Including higher dimensions into consideration results only in some scaling of the coupling parameter η, which may be calculated by means of geometrical optics since it characterizes the distance between the WKB cutoff surfaces. However, the entire process is very similar to the one-dimensional conversion.

However, the situation discussed above is not general. Based on Maxwell's equations, one can show that in a region of effective conversion the geometro-optical coupling constant vanishes, $\eta\to 0$, thus it must be replaced by a differential operator. This results in an essentially non-one-dimensional character of wave interaction that cannot be described by techniques developed in [4–6]. This case was considered by Weitzner, who discovered that conversion of the electron cyclotron waves in a nonplanar tokamak plasma occurs in an essentially wider range of incident beam parameters as compared to one-dimensional predictions [7]. However, solutions obtained by Weitzner could hardly be applied to describe a realistic case since they are not matched to any incident beam and contain singularities. A theory free from singularities was developed by Gospodchikov and coworkers [8–10] and by Popov and Piliya [11] in the particular case of cyclotron wave coupling in a two-dimensionally magnetized plasma.

Below we present an extension of this theory to a three-dimensionally inhomogeneous medium with a dispersion matrix of rather general form. We consider a gyrotropic medium without spatial dispersion and dissipation. The medium is assumed to be weakly inhomogeneous on the wavelength scale, but the axis of gyrotropy has a constant direction in the region of mode coupling. Also we guess that there are no resonances with the medium, i.e., the wavelength is small but not going to zero. Thus we exclude from consideration the cases of sheared gyrotropy in a weakly anisotropic medium [2,12] and coupling to "hot" kinetic modes in the vicinity of the resonances [1]. With these two restrictions, our case seems to be the only nontrivial configuration in free space in which mode coupling occurs [13].

In the WKB approximation Maxwell's equations are reduced to the algebraic equations $D_{ij}E_j=0$, with $D_{ij}=k^2\delta_{ij}-k_ik_j-k_0^2\varepsilon_{ij}$ being the standard dispersion matrix, $E_j\propto\exp(i\mathbf{k}\cdot\mathbf{r}-i\omega t)$, k_j being the components of the electric field and wave vector of a local plane wave in some Cartesian coordinates x_1, x_2, x_3, and $k_0=\omega/c$. The electric field may be decomposed over three arbitrary polarizations by unitary transformation $\mathbf{U}$:

$$\mathbf{E}=\mathcal{E}_1\mathbf{e}_1+\mathcal{E}_2\mathbf{e}_2+\mathcal{E}_3\mathbf{e}_3\equiv\mathbf{U}\boldsymbol{\mathcal{E}},\quad \boldsymbol{\mathcal{E}}=\mathbf{U}^{\dagger}\mathbf{E};$$

here U_{ij} is the Cartesian x_j coordinate of the polarization vector $\mathbf{e}_i$ and $\dagger$ denotes the Hermitian conjugate. Since the dielectric tensor ε_{ij} is Hermitian, it may be diagonalized by a particular choice of the polarization vectors. Then, Maxwell's equations for the components $\mathcal{E}_i$ are

$$\mathcal{D}_{ij}\mathcal{E}_j=0,\quad \mathcal{D}_{ij}=(k^2-k_0^2\varepsilon_i)\delta_{ij}-U_{im}k_mk_nU_{jn}^*,\tag{1}$$

with $\mathcal{D}_{ij}=U_{im}D_{mn}U_{jn}^*$ and $\varepsilon_i=U_{im}\varepsilon_{nm}U_{in}^*\in\mathbb{R}$; no summation over i is assumed. Let us place the coordinate x_3 along the axis of gyrotropy; then ε_{13}, ε_{23}, ε_{31}, and ε_{32} are zero, and the matrix of polarization vectors, in which the dielectric tensor is diagonal, takes the following form:

$$\mathbf{U} = \begin{bmatrix} \cos\gamma & \sin\gamma & 0 \\ -i\sin\gamma & i\cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\delta & \sin\delta & 0 \\ -\sin\delta & \cos\delta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (2)$$

with δ and γ defined by the dielectric tensor ε_{ij}. With transformation to components $\mathcal{E}_i$, we describe the electric field as a superposition of two elliptic polarizations $\mathbf{e}_1$ and $\mathbf{e}_2$ orthogonal to each other and to the axis of gyrotropy, and linear polarization $\mathbf{e}_3$ along the axis of gyrotropy; γ defines the "ellipticity" of transverse polarizations, e.g., $\gamma=0,\pi$ for linear polarizations, $\gamma=\pm\pi/4$ for circular polarizations (Stix frame); δ defines the inclination of the polarization ellipses in the x_1,x_2 plane. Without loss of generality we assume below that $\delta=0$.

From previous analysis it is known that effective mode coupling in an inhomogeneous medium is possible in the vicinity of the reflection point where $k_1,k_2 \ll k_3$ [1,14]. Indeed, for strictly longitudinal propagation, $k_1=k_2=0$, the dispersion equation $\det(\mathcal{D}_{ij})=0$ takes the form

$$\varepsilon_1'\varepsilon_2'\varepsilon_3=0, \quad \varepsilon_{1,2}'=\varepsilon_{1,2}-k_3^2/k_0^2,$$

from which it follows that in the vicinity of the reflection point at least one of the three multipliers ε_1', ε_2', and ε_3 is small. Mode coupling occurs when the reflection points for two modes converge, i.e., two of three multipliers are small. For definiteness, below we consider

$$|\varepsilon_1'| \ll 1, \quad |\varepsilon_3| \ll 1, \quad k_1 \ll k_3, \quad k_2 \ll k_3. \quad (3)$$

These conditions define the coupling of two modes with polarizations $\mathcal{E}_1/\mathcal{E}_3 \approx \pm k_3(k_1\cos\gamma+ik_2\sin\gamma)/(k_0^2\varepsilon_1')$ and $\mathcal{E}_2 \ll \mathcal{E}_1,\mathcal{E}_3$. The case of $|\varepsilon_2'|,|\varepsilon_3| \ll 1$ may be studied in the same manner; the case of $|\varepsilon_1'|,|\varepsilon_2'| \ll 1$ corresponds to $\mathcal{E}_1,\mathcal{E}_2$ polarization degeneracy in a weakly anisotropic medium, which is excluded from consideration.

Taking into account the conditions of coupling (3), one can obtain the following dispersion relation:

$$k_1^2\cos^2\gamma+k_2^2\sin^2\gamma=k_0^4\varepsilon_1'\varepsilon_3/k_3^2. \quad (4)$$

Formally, this dispersion equation describes two modes separated by an evanescent region defined by the condition $\varepsilon_1'\varepsilon_3<0$. The mode conversion appears as tunneling of radiation throughout the evanescent region. In the one-dimensional case this region lies between two plane-parallel cutoff surfaces $\varepsilon_1'=0$ and $\varepsilon_3=0$, and the coupling efficiency depends exponentially on the distance between the cutoff surfaces; see, e.g., [1,4,14]. For example, perfect mode conversion (with no reflection) occurs in the degenerate case when both cutoff surfaces coincide at a certain propagation angle, and the evanescent region is then absent [14].

However, the problem becomes topologically different in the two- or three-dimensional case where the cutoff surfaces intersect as shown in Fig. 1 [15]. There is no evanescent region for a ray passing through the intersection line between the cutoff surfaces, which formally corresponds to the case of perfect conversion. In contrast to the one-dimensional model, this case is not degenerate because moderate variation of the propagation angle results only in a shift of the intersection line in space. Of course, in the vicinity of the coupling region the WKB approach becomes invalid, but analysis of the inhomogeneous wave equations presented below yields essentially the same result.

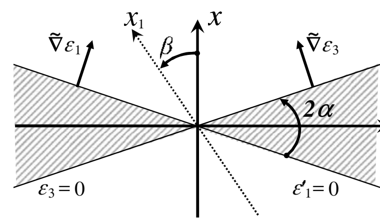

FIG. 1. Cutoff surfaces in non-one-dimensional geometry and coordinate system used in the reference wave equations (6). The evanescent region for the coupled waves is hatched.

Equations for the coupled waves may be derived by substituting $k_{1,2}\to -i\partial/\partial x_{1,2}$, $k_3\to k_3-i\partial/\partial x_3$ in Eq. (1), and canceling all second derivatives, assuming that the medium is smoothly inhomogeneous, $k_0L\gg 1$, for all scales L of spatial variation of ε_{ij}. Due to the constant axis of gyrotropy, with the same accuracy one may ignore terms resulting from spatial variation of the polarization matrix (2). Finally, we obtain the following equations:

$$ik_3(\cos\gamma\,\partial/\partial x_1+i\sin\gamma\,\partial/\partial x_2)\mathcal{E}_3=(k_0^2\varepsilon_1'+2ik_3\partial/\partial x_3)\mathcal{E}_1,$$
$$ik_3(\cos\gamma\,\partial/\partial x_1-i\sin\gamma\,\partial/\partial x_2)\mathcal{E}_1=k_0^2\varepsilon_3\mathcal{E}_3. \quad (5)$$

With the same accuracy $\mathcal{E}_2=0$, what justifies the assumption that the mode with non-negligible $\mathcal{E}_2$ is decoupled.

Spatial inhomogeneity is revealed in Eqs. (5) as the variation of only two parameters, ε_1' and ε_3, which allows us to reduce the generally three-dimensional case to a two-dimensional problem. Having in mind that within the coupling region (characterized by size $L_\nabla \sim L/\sqrt{k_0L} \ll L$) parameters of the medium vary weakly, we retain only linear variations, $\varepsilon_1'=\boldsymbol{\nabla}\varepsilon_1\cdot\mathbf{r}$ and $\varepsilon_3=\boldsymbol{\nabla}\varepsilon_3\cdot\mathbf{r}$, i.e., we consider intersection of *flat* cutoff planes. In this approximation, the problem may be treated as locally two-dimensional with an axis of homogeneity directed transverse to both $\boldsymbol{\nabla}\varepsilon_1$ and $\boldsymbol{\nabla}\varepsilon_3$. We assume for simplicity that this axis coincides with the axis of gyrotropy x_3, but later we will show that the general case may be studied analogously.

When $\sin 2\gamma=0$, Eqs. (5) describe mode coupling in the absence of gyrotropy. This trivial case is reduced to a one-dimensional problem that will not be considered below. In all other cases, Eqs. (5) may be reduced to the following canonical form:

$$-(i\partial/\partial x-\partial/\partial y)A_3=(x\cos\alpha+y\sin\alpha)A_1,$$
$$-(i\partial/\partial x+\partial/\partial y)A_1=(x\cos\alpha-y\sin\alpha)A_3. \quad (6)$$

We introduce new dimensionless coordinates symmetrized with respect to the cutoff surfaces (see Fig. 1),

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos\beta/\cos\gamma & \sin\beta/\sin\gamma \\ -\sin\beta/\cos\gamma & \cos\beta/\sin\gamma \end{bmatrix} \begin{bmatrix} x_1/L_\nabla \\ x_2/L_\nabla \end{bmatrix},$$

and normalized components of the electric field

$$A_1=-\exp(i\beta)|\tilde{\boldsymbol{\nabla}}\varepsilon_1|^{-1/2}\mathcal{E}_1, \quad A_3=|\tilde{\boldsymbol{\nabla}}\varepsilon_3|^{-1/2}\mathcal{E}_3.$$

The x and y axes are directed along the bisectors between $\tilde{\boldsymbol{\nabla}}\varepsilon_1$ and $\tilde{\boldsymbol{\nabla}}\varepsilon_3$; the vectors $\tilde{\boldsymbol{\nabla}}\varepsilon_{1,3}=\hat{\mathbf{x}}_1(\partial\varepsilon_{1,3}/\partial x_1)\cos\gamma+\hat{\mathbf{x}}_2(\partial\varepsilon_{1,3}/\partial x_2)\sin\gamma$ define the gradients in the new coordinates; the angles α and β are, respectively, the half angle between the cutoff surfaces in the x,y space and the angle between the x and x_1 axes; and $L_\nabla = k_3^{1/2} k_0^{-1} (|\tilde{\nabla}\varepsilon_1||\tilde{\nabla}\varepsilon_3|)^{-1/4}$ is the characteristic length of the coupling region.

The reference wave equations (6) may be solved analytically. First, considering the WKB limit $|x| \gg 1$, one can determine the polarization of waves propagating along the x axis as $A_1/A_3 = \pm 1$. Therefore, an arbitrary wave field may be represented as $A_1 = A^+ + A^-$, $A_3 = A^- - A^+$, where $A^\pm$ corresponds to waves that propagate in the WKB region in two opposite directions and have asymptotics $A^\pm \to \exp(\mp i x^2/2) \cos \alpha$ when $|x| \to \infty$. With new field definitions, Eqs. (6) are transformed to

$$-(i\partial/\partial x + x \cos \alpha) A^- = (\partial/\partial y + y \sin \alpha) A^+,$$

$$-(i\partial/\partial x - x \cos \alpha) A^+ = (\partial/\partial y - y \sin \alpha) A^-. \qquad (7)$$

Applying the operators $i\partial/\partial x \pm x \cos \alpha$, one may decouple the equations for A^+ and A^- in a form that allows solution by separation of variables. Assuming $A^\pm = A_x^\pm(x) A_y^\pm(y)$, one derives the set of ordinary differential equations

$$(-\partial^2/\partial x^2 + x^2 \cos^2 \alpha \mp i \cos \alpha) A_x^\pm = \mu A_x^\pm, \qquad (8a)$$

$$(\partial^2/\partial y^2 - y^2 \sin^2 \alpha \pm \sin \alpha) A_y^\pm = \mu A_y^\pm. \qquad (8b)$$

Solutions of both equations are given by the parabolic cylinder functions $D_\nu(z)$. Let us consider the wave structures localized inside the evanescent region: $A_y^\pm \to 0$ when $y \to \pm\infty$. Then, Eq. (8b) reduces to the problem of a quantum harmonic oscillator with a discrete spectrum $\mu = -\sin|\alpha| (2n+1 \pm \mathrm{sgn}\, \alpha)$ with $n = 0, 1, \ldots$. Equation (8a) is essentially the same as in a slab problem homogeneous in the y direction, except that a discrete spectrum appears instead of the continuous spectrum $\mu = -k_y^2 L_\nabla^2$ typical of the one-dimensional case [14].

The particular choice of a solution is defined by its asymptotic behavior in the WKB region. The direction of wave propagation is determined by the angle between the group velocity and wave vector; more exactly, by the sign of $\partial(k_0^2 \varepsilon_1' \varepsilon_3)/\partial\omega$ [this follows from the dispersion relation (4)]. Therefore, $A^\pm$ may define waves propagating in either the positive or the negative direction along the x axis. For definiteness, we consider that A^+ and A^- stand, respectively, for waves propagating toward $x \to +\infty$ and $x \to -\infty$. Then, for the radiation launched from $-\infty$, the field A^+ describes the incident and transmitted waves; the field A^- describes the reflected wave when $x \to -\infty$ and vanishes when $x \to +\infty$. The corresponding solution of Eqs. (7) is found in our previous work [8,9]:

$$A^+ = \sum_{n=0}^{\infty} A_n \tilde{D}_{i\nu_n}(x) h_n(y),$$

$$A^- = \sum_{n=0}^{\infty} \begin{cases} A_{n+1}\sqrt{i\nu_{n+1}}\tilde{D}_{i\nu_{n+1}-1}(x) h_n(y), & \alpha > 0, \\ -A_n \sqrt{i\nu_n}\tilde{D}_{i\nu_n - 1}(x) h_{n+1}(y), & \alpha < 0. \end{cases} \qquad (9)$$

Here, A_n are arbitrary constants,

$$\nu_n = (n + 1/2 - \mathrm{sgn}\, \alpha/2) \tan|\alpha|, \qquad (10)$$

$\tilde{D}_{i\nu}(x) = D_{i\nu}[\exp(i\pi/4)\sqrt{2\cos\alpha}\, x]$ describes the propagation of a wave beam, and $h_n(y) = \pi^{-1/4}(n!)^{-1/2} \times \sin^{1/4}|\alpha| D_n(\sqrt{2\sin|\alpha|}\, y)$ describes its transverse structure. The functions $h_n(y)$, which may be alternatively expressed via Hermite polynomials, form an orthogonal basis; therefore, A_n may be derived as projections of the incident wave,

$$A_n = \tilde{D}_{i\nu_n}^{-1}(x) \int_{-\infty}^{+\infty} A^+(x,y) h_n(y) dy.$$

Substituting this definition in Eqs. (9), one can recover the field distribution in the transmitted and reflected waves for a given incident beam.

The coupling efficiency may be defined as the ratio of total power fluxes in the incident wave beam and that passed through the interaction region, $T = I_+/I_-$, where $I_\pm = \int |A^+(x \to \pm\infty)|^2 dy$. Using the WKB asymptotics of solution (9) and Parseval's theorem for orthogonal series, one obtains

$$T = \frac{1}{P}\sum_{n=0}^{\infty} |A_n|^2 \exp(-2\pi\nu_n), \quad P = \sum_{n=0}^{\infty} |A_n|^2. \qquad (11)$$

As shown in [9], one may perform a summation over n analytically for any field distribution of the incident wave. For instance, for an incident Gaussian beam $A^+ \propto \exp(-y^2/2a^2)$, the coupling coefficient is $T = (1+\varphi)(1+\varphi a^2/\sin\alpha)^{-1/2}(1+\varphi \sin\alpha/a^2)^{-1/2}$ where $\varphi = \tanh(\pi \tan \alpha)$.

Let us focus on physical effects that are absent in one-dimensional and "local coupling" theories [6]. The first effect is related to modification of the reciprocity law in gyrotropic media that result in the absence of reversibility with respect to change of the propagation direction. Indeed, the reverse of the propagation direction along the x axis is equivalent to interchange of the two cutoff surfaces shown in Fig. 1; the latter is described by a change of sign of the angle α. Different signs of α result in different spectra (10), and therefore in different coupling efficiencies (11). One obtains that

$$T(\alpha) = T(-\alpha)\exp(2\pi \tan \alpha).$$

Thus, the coupling is more effective for propagation along the x axis from $-\infty$ to $+\infty$ when $\alpha > 0$, and in the opposite direction when $\alpha < 0$; in the one-dimensional case, $\alpha = 0$, the reciprocity law is recovered in its usual form.

Another feature of two-dimensional geometry is the existence of optimal wave beams which exhibit perfect mode conversion without reflection. Such structures may be found from the solution (9) in which only the $n=0$ term is present in the sum and, additionally, $\nu_0 = 0$. In this case, the optimal wave distribution is given by

$$A^\pm = C \exp[\mp (ix^2/2)\cos\alpha - (y^2/2)\sin|\alpha|], \quad A^\mp = 0, \qquad (12)$$

the upper and lower signs corresponding to $\alpha > 0$ and $\alpha < 0$. Thus, Gaussian beams of a specific width pass through the two-dimensional coupling region with no modification. This differs from the one-dimensional result that only a plane wave with a certain parallel wave vector k_3 may pass through a slab interface without reflection [14]. The absence of a unique value of k_3 corresponding to perfect conversion is an essentially two-dimensional effect—a small variation of k_3 would result only in displacement of the intersection line of the cutoff surfaces; therefore, simultaneous variation of k_3 and proper displacement of the x,y coordinates in (12) also result in a perfectly converting beam.

All the above results correspond to wave structures with fixed k_3. One can consider structures limited in the x_3 coordinate as a superposition of different k_3 harmonics; however, one should keep in mind the displacement of the local coordinates for different k_3. In this way it is found that the form of two-dimensional Gaussian beams is restored after passing the mode conversion region, although the parameters of the transmitted Gaussian beam are modified [9]. Perfectly converted beams may be realized with astigmatic two-dimensional Gaussian beams with phase modulation [10].

Now we return to the general case when the medium is inhomogeneous along the axis of gyrotropy x_3, but all spatial variations are still linear. Then, new terms proportional to $\lambda_{1,3}=|\tilde{\nabla}\varepsilon_{1,3}|^{-1}\partial\varepsilon_{1,3}/\partial x_3$ appear in the right-hand side (RHS) of Eqs. (6) (the LHS is unchanged):

$$\cdots = (x\cos\alpha + y\sin\alpha + \lambda_1 z + i\kappa\partial/\partial z)A_1,$$

$$\cdots = (x\cos\alpha - y\sin\alpha + \lambda_3 z)A_3, \tag{13}$$

where $z=x_3/L_\nabla$ and $\kappa=\sqrt{|\tilde{\nabla}\varepsilon_3|/|\tilde{\nabla}\varepsilon_1|}$. Substituting

$$A_{1,3}\exp[i(\lambda_1-\lambda_3)z^2/(2\kappa)], \quad x\cos\alpha + \lambda_3 z \to x'\cos\alpha,$$

Eqs. (13) are transformed to

$$\cdots = (x'\cos\alpha + y\sin\alpha + i\eta\partial/\partial x' + i\kappa\partial/\partial z)A_1,$$

$$\cdots = (x'\cos\alpha - y\sin\alpha)A_3,$$

$\eta=\lambda_3\kappa/\cos\alpha$. Since these equations are homogeneous over the z coordinate, the term $\partial/\partial z$ may be taken into account by redefinition of k_3. Thus, we again arrive at two-dimensional wave equations. When $\lambda_3=0$, which naturally happens, e.g., in tokamak plasmas, the obtained equations reduce to Eqs. (6) studied above. The generalization for $\lambda_3\neq 0$ will be considered in a separate presentation.

The transition from the two- to the one-dimensional case was investigated in [8]. The slab approximation is applicable when $\alpha\ll 1$ and $\alpha^2 a_y^2/L_\nabla^2\ll 1$, where a_y is the characteristic width of the beam along the y axis. The latter condition is equivalent to a non-negligible variation of the one-dimensional coupling efficiency within the beam aperture. For sufficiently wide beams this condition may not be valid; then the two-dimensional description is required even when $\alpha\ll 1$. Note that two-dimensional effects may be pronounced even when the angle between the cutoff surfaces (between $\nabla\varepsilon_1$ and $\nabla\varepsilon_3$) is small but not zero, i.e., in a case that seems to be one-dimensional. Indeed, the parameter 2α is defined as the angle between $\tilde{\nabla}\varepsilon_1$ and $\tilde{\nabla}\varepsilon_3$ which depends on the polarization of the waves. This angle tends to increase for waves polarized elliptically with elongation along $\nabla\varepsilon_1-\nabla\varepsilon_3$; however, the case of strictly linear polarization is always one-dimensional.

The effects found within the two-dimensional theory of mode coupling seem to be rather universal; thus the presented results may be considered as a solution of a standard problem in electrodynamics.

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